Modeling and Forecasting Zimbabwe’s Trade Exports with Time Series Analysis

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Trade is one of the most important factors in a country’s development. A country’s level of exports indicates the strength of its manufacturing and entrepreneurial capabilities, as well as the degree to which it is harnessing its natural resources. For transitional countries especially, increased trade is essential if they are to break free from the traps of poverty and construct a more stable and prosperous economy. Given the increasing importance of trade analysis, statistics proves essential to this topic and is beginning to be utilized in fields as diverse as international development, public policy, and political science.

For this project, I used statistical time series methods to analyze, model and then forecast the trade export data from Zimbabwe. According to an article published by the National Institute of Standards and Technology in partnership with the U.S. Department of Commerce, a time series is “An ordered sequence of values of a variable at equally spaced time intervals…[It] accounts for the fact that data points taken over time have an internal structure (such as autocorrelation, trend or seasonal variation) that should be accounted for” [2]. Time series is consequently a powerful tool for analyzing any data set in which the observations for the variable of interest somehow relate to a chronological or time-based unit. Examples of data sets that could be characterized as time series are a department store’s quarterly revenue over the last
five years or Mexico’s monthly rainfall over the past decade. For both of these data sets, the output variable is invariably linked to the time at which the data was recorded.

The data I used to model Zimbabwe’s trade exports came from the online database, International Trade Statistics, which is moderated by the International Trade Centre located in Geneva, Switzerland. The data spans from January 2010 to December 2016 and is broken up on a monthly basis. All data was converted to the United States dollar based on the mean, interbank currency rating as published by fxtop.com. It follows the standard unit of USD thousand. A graph of the raw data can be seen in Figure 1 of the Appendix.

When analyzing data in light of time series methodology, it is first important to understand overall patterns within the data set, such as whether the data exhibits an upward or downward trend, or has a non-constant mean/variance over time. Such questions can be generally answered by determining whether the data is stationary or nonstationary. Stationarity requires that a time series’ statistical characteristics do not change in relation to time. Stationary time series therefore exhibit a constant mean and standard deviation, no trend behavior and a joint distribution that is constant over time. In their book *Time Series Analysis with Applications in R*, Cryer and Chan explain that “a process \( \{Y_t\} \) is said to be strictly stationary if the joint distribution of \( Y_{t_1}, Y_{t_2}, ..., Y_{t_n} \) is the same as the joint distribution of \( Y_{t_1-k}, Y_{t_2-k}, ..., Y_{t_n-k} \) for all choices of time points \( t_1, t_2, ..., t_n \) and all choices of time lag \( k \),” where a lag is defined as some designed interval of time [3, p. 16]. Therefore if one were to observe random snapshots of the data at different times, we would anticipate relatively similar behavior when analyzed in terms of the overall series [1, p. 232]. However, given these constraints it should come as no surprise that almost all economic and business related data is nonstationary. We expect that the revenue associated with most properly run businesses will follow a steadily increasing trend instead of
oscillating around a fixed mean. Similarly, we expect a country’s trade exports to increase during times of economic prosperity and to plummet in response to natural disasters or economic hardships. As a result, very few economic type data sets can be properly modeled as stationary time series.

Considering my data of Zimbabwe’s trade exports, it is tempting to assume that it is nonstationary because of its slightly fluctuating standard deviation and mean values. For example, assuming that a time period consists of a single calendar year, we find that the mean value of trade exports in 2013 is approximately 63,787 as compared to 2016’s average of 84,094. Furthermore, the standard deviation associated with 2016 is 236,023 and that for 2012 is roughly 323,623. Yet several tests can be used to quantitatively evaluate whether or not a time series is nonstationary with one of these methods being the Dickey-Fuller Unit Root Test.

The Dickey-Fuller Test was introduced by statisticians David Dickey and Wayne Fuller in the late 1970s to mathematically evaluate the stationary or nonstationary behavior of a time series. This test starts by considering a model of the form \( Y_t = \alpha Y_{t-1} + X_t \), where it is assumed that \( \{X_t\} \) is stationary. As a result, we see that the stationary behavior of the process \( \{Y_t\} \) must depend on the coefficient \( \alpha \), i.e. if \( \alpha = 1 \) than \( \{Y_t\} \) is nonstationary and if \(-1 < \alpha < 1\) than \( \{Y_t\} \) is stationary. If we then assume that the \( \{X_t\} \) follows an AR(p) process (I will delve into this topic more deeply later in the paper) of the form \( X_t = \phi_1 X_{t-1} + \cdots + \phi_k X_{t-k} + \varepsilon_t \), assume that \( \alpha = 1 \) and algebraically manipulate the variables, we find that

\[
X_t = Y_t - Y_{t-1} = \alpha Y_{t-1} + \phi_1 (Y_{t-1} - Y_{t-2}) + \cdots + \phi_k (Y_{t-k} - Y_{t-k-1}) + \varepsilon_t.
\]

If, however, \(-1 < \alpha < 1\), then we again obtain an equation like that presented above, except for a change of coefficients. As a result, \( \{Y_t\} \) can be categorized as an AR characteristic polynomial. The null hypothesis for the Dickey-Fuller Test is therefore that this polynomial will have a unit root.
versus the alternative statement that no such roots exist. Such a formulation is synonymous to stating the null and alternative hypothesis in terms of stationary and nonstationary behavior. Namely, our null hypothesis assumes that our time series is nonstationary or that $\alpha = 1$, which can be shown to be algebraically equivalent to $a = 0$ in our modified equation above. Our alternative hypothesis, $a < 0$, assumes stationarity [3, p. 128].

Although the Dickey-Fuller Test often seems cumbersome to understand especially in terms of how it might be performed, it can be generally understood in the following way. Whenever we have nonstationary data, it is possible to differentiate the series in order to obtain a modified time series that is indeed stationary. This is what we want if we are to find an appropriate process that will model and then forecast our data. The Dickey-Fuller Test takes advantage of this notion by evaluating whether under the null hypothesis, the time series becomes stationary after differentiation versus the alternative hypothesis that the series is already stationary and thus no differentiating is required [3, p. 129]. Thankfully, the Dickey-Fuller Test can be performed on most statistical software packages and thus requires only an overall understanding of its key principles.

Yet, it is important to note that because the Dickey-Fuller Test is constructed with assuming nonstationary behavior in the null hypothesis, we cannot proceed with a standard $t$ test, as the assumptions underlying the normal distribution would be violated. As a result, this test calculates a test statistic $\tau$, which we will then compare to the appropriate critical value from the Dickey-Fuller Table. If $\tau$ is less than the critical value, we can conclude a significant result.

In order to evaluate the data associated with Zimbabwe’s trade exports, I downloaded the statistical package XLSTAT, which can be used in conjunction with Excel. (Note that Minitab does not perform this test, and although XLSTAT is not the most ideal statistical software to
employ in this setting, it was the only one available to me). Using an alpha of 0.05, I obtained a test statistic of -4.517 as compared to the critical value of -0.774. The resulting p-value was 0.002. Because my test statistic was significant, I have enough evidence to conclude that my time series is stationary, and will therefore proceed under this assumption. The risk of rejecting the null hypothesis when it was indeed true is less than 0.21%.

In addition to the Dickey-Fuller Test, we can also evaluate the graph of the autocorrelation function (ACF) and the graph of the partial autocorrelation function (PACF) to determine stationary versus nonstationary behavior. The autocorrelation function measures the degree of correlation between all the data values $y_t$ and $y_{t+k}$ which are $k$ intervals apart from one another. As stated above, this interval $k$ can be defined as the lag. Therefore in terms of our problem, the autocorrelation function will determine the correlation between data points that are one month apart, two months apart, three months apart, etc. The autocorrelation coefficient at lag $k$ can thus be expressed by the formula,

$$
\rho_k = \frac{\mathbb{E}[(y_t-\mu)(y_{t+k}-\mu)]}{\sqrt{\mathbb{E}[(y_t-\mu)^2]\mathbb{E}[(y_{t+k}-\mu)^2]}} = \frac{\text{Cov}(y_t,y_{t+k})}{\text{Var}(y_t)},
$$

where the collection of values $p_k$, such that $k = 0, 1, 2, \ldots$, constitutes the autocorrelation function. This function is both symmetric around zero and dimensionless, meaning that it is quantified independently from the measurement scale of our given data [1, p. 30]. Figure 2 of the Appendix shows the graph of the autocorrelation function for our data set. Observing our graph, we can note significant correlation between data points that are one month and two months apart, as well as an almost significant correlation at the twelve month or one year lag.

Similar to the autocorrelation function, the partial autocorrelation function measures the degree of correlation, at lag $k$, between the data values which are $k$ units apart from one another, while also taking into account the observations that occur between our intervals of interest. Montgomery, Jennings, and Kulahci define the partial autocorrelation function between $y_t$ and
$y_{t-k}$ as the “autocorrelation between $y_t$ and $y_{t-k}$ after adjusting for $y_{t-1}, y_{t-2}, \ldots, y_{t-k+1}$” [1, p. 249]. When assuming normality, the partial autocorrelation function at lag $k$ can therefore be mathematically represented by $\phi_{kk} = Corr(Y_t, Y_{t-k} | Y_{t-1}, Y_{t-2}, \ldots, Y_{t-k+1})$ or the correlation between $Y_t$ and $Y_{t-k}$ given $Y_{t-1}, Y_{t-2}, \ldots, Y_{t-k+1}$. If, however, we do not assume that $\{Y_t\}$ is normal, then the partial autocorrelation function becomes $\phi_{kk} = Corr(Y_t - \beta_1 Y_{t-1} - \beta_2 Y_{t-2} - \cdots - \beta_{k-1} Y_{t-k+1}, Y_{t-k} - \beta_1 Y_{t-k+1} - \beta_2 Y_{t-k+2} - \cdots - \beta_{k-1} Y_{t-1})$, where each $\beta$ value is strategically selected to minimize our mean square prediction error [3, p. 112-113]. Figure 3 in the Appendix shows the graph of the partial autocorrelation function for Zimbabwe’s export data. Both the autocorrelation function and the partial autocorrelation function can be interpreted using standard correlation techniques, where the correlation ranges from a scale of $-1$ to $1$ with $-1$ indicating strong, negative correlation, $0$ no correlation and $1$ strong, positive correlation.

To confirm whether our time series is stationary or nonstationary, the behavior of the autocorrelation function proves particularly useful. The ACF of stationary sequences usually begins with one or two significant lags and then experiences exponential decay, with the remainder of the lags roughly hovering around zero and demonstrating sinusoidal behavior. The ACF of non-stationary data, on the other hand, experiences very strong and gradual decay, thus revealing a large number of significant lags. When looking at our ACF graph, we observe two significant lags at $k=1, 2$, after which the lags quickly decay and demonstrate sinusoidal behavior. Later, we will observe how the PACF graph can be utilized for model selection, but for now we will simply conclude that the autocorrelation function confirms the results generated from the Dickey-Fuller Test, and we can therefore proceed under the assumption of stationary behavior.
As stated above, such results should be alarming for economic data as we expect most real-life time series to display nonstationary behavior. But stationary behavior for developing and transitional countries’ trade exports should be extra surprising for at least one major reason. The first reason stems from the fact that developing countries’ economies are much more volatile and are often exposed to negative factors such as famine, war, social unrest, and political corruption. All of these factors produce sharp negative trends that take years and sometimes decades to stabilize, thereby triggering nonstationary behavior. On the other hand, transitional countries have mostly overcome such severe economic hardships and are experiencing accelerated economic growth relative to their historical performance. Therefore, the value of trade exports associated with transitional countries often displays some form of a positive trend.

For our data set, the fact that Zimbabwe’s exports seem to be stagnant over the last seven years should raise red flags and point to fundamental weaknesses within Zimbabwe’s economy. Zimbabwe’s unemployment rate, for instance, skyrocketed to 95% in 2009, just one year before our data was collected, and Zimbabwe has generally ranked as one of the poorest countries in the world. Such stationary behavior within the trading sector could therefore be attributed to Zimbabwe’s inability to escape its harsh economic conditions and suggests an overall inability to get back on its feet. Such investigation, however, goes beyond the scope of this paper and could be tackled as a future topic of interest.

After testing for and assuming stationary behavior, the next step is to model our data. Using the results of the Dickey-Fuller Test, we will use models that are specifically designed to represent stationary time series. The two major options for modeling such data are the MA and AR models.
The MA (or moving average) model is useful when we want to utilize past forecast errors in order to construct a formula similar to a regression-type model. MA models are structured so as to emphasize a more finite and smaller number of past forecast errors, instead of taking into account a series’ overall (and more lengthy) past behavior. For instance an MA model of order 2 would model each successive term with a weighted average of the last two preceding values, an MA model of order 3 would use a weighted average of the last three values, etc. The MA model can therefore be understood as a weighted moving average that is based on a selected number of previous forecast errors. As a result, the formula for each observation $y_t$ for a moving average process of order $q$, denoted MA($q$), is $y_t = \mu + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \cdots - \theta_q \varepsilon_{t-q}$, where $q$ represents the lag after which all lags tend towards zero and are insignificant. In this model, we can see that $\mu$ signifies our process mean or constant variable and $\{\varepsilon_t\}$ is white noise [1, p.235]. We define white noise as a sequence consisting of uncorrelated observations and some constant variance and mean, where the mean is generally assumed to be zero. Because white noise possesses uncorrelated observations, we typically cannot use previous values to forecast upcoming results. Since Zimbabwe’s trade exports fall under the larger umbrella of economic data, however, we can understand white noise as shock, or namely those points in a time series, which are unpredictable and whose results catch us off guard. Finally, the variables $\theta_1, \theta_2, \ldots, \theta_q$ are the weighted coefficients. Adjusting these coefficients may drastically alter the pattern of the generated time series. For this paper, I assumed negative coefficients since many statistical software packages are programmed in this manner [1, p. 2-3]. Many texts, however, use the related formula that is identical to the one above, except that it assumes positive coefficients. (The formula you select should align with whatever statistical program you choose to employ).
We can determine an approximate order of our MA($q$) model by evaluating the behavior of our ACF and PACF graphs. For MA($q$) models, the ACF graph must display non-zero values for the first $q$ lags with the remainder of values tending towards zero. The PACF graph must simply tend towards zero in some unspecified pattern. Looking once more at our ACF graph found in Figure 2 of the Appendix, we can observe spikes at the first two, maybe three, lags with the remainder of lags tending towards zero. (It is important to note, however, that an almost significant lag appears at lag 12, which reveals some degree of inconsistency within our data). Thus, we can first select an MA(2) or MA(3) model, but should also try moving average processes of other orders to ensure that we are indeed selecting the most appropriate one.

However, before settling on an MA($q$) model, it is important to at least be aware of and consider other modeling processes that may be useful. One such model is the autoregressive process of order $p$, (AR($p$)), which is simply a regressive model (i.e. a model testing the relationship between variables, say, the independent and dependent variable) that is regressive with itself. As hinted at in the equations earlier, the moving average process assumes that, as time progresses, the effect of observations outside of some designated interval is obsolete. Thus current terms are mostly influenced by the observations immediately preceding them. Yet, it may be the case that the observations in our model are indeed influenced by the more lasting impacts of terms stretching farther into the past. Nonetheless even if past observations do exert a lingering effect, it seems reasonable to assume that observations recently occurring are much more influential than those that occurred months or even a year ago [1, p. 239]. Taking this into account, AR($p$), or the $p^{th}$-order autoregressive process $\{Y_t\}$ can be expressed by the equation $Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + e_t$, where the coefficients $\phi_1$, $\phi_2$, $\phi_3$ are weighted more heavily the closer they get to our term of interest. Hence the current value of our series, $Y_t$, is
calculated according to the previous $p$ observations, where $e_t$ takes into account everything not explained by these past $p$ terms. As a result, $e_t$ is required to be independent of the past $p$ values, i.e. $Y_{t-1}, Y_{t-2}, Y_{t-3} \ldots$ [3, p. 66]. Furthermore, the order of an AR($p$) process can again be determined by inspecting the ACF and PACF for our data. The PACF of an AR($p$) process will reveal non-zero values up until lag $p$, with the remaining values tending towards zero. The ACF must simply tend towards zero in some unspecified pattern.

Combining the notions underlying both moving average and autoregressive processes, the autoregressive moving average process of order $p$ and $q$, denoted ARMA($p,q$), intertwines both of these notions. An ARMA($p,q$) process therefore accounts for both previous observations and past errors. In addition, when using an AR($p$) process, it may not be possible to utilize an exponential decay pattern to approximate our weights without resorting to a higher order AR($p$) process. The downfall to this approach is that it may add exceedingly many terms to our model, when the problem could possibly be corrected by simply adding an MA($q$) component that results in an overall model with many fewer terms. As a result, \( \{Y_t\} \) is an ARMA($p,q$) process if \[ Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}. \] It is important to note that an ARMA($p,0$) model is equivalent to AR($p$), whereas an ARMA($0,q$) is equivalent to MA($q$).

One way to determine which model to select is by comparing the residual sum of squares generated by each process. Models that better represent the data set will naturally have smaller residual sum of squares than those that do not. In Table 2 of the Appendix, I listed many possible processes with their respective residual sum of squares and suggested models. All data was generated from Minitab and did not include a constant term in the model. After reviewing this
table, we can quickly observe that the models with the lowest residual sum of squares are those which contain an MA(4) component, i.e. ARMA(1,4), ARMA(2,4), and ARMA(4,4).

Yet there exists a major flaw in these models that up until now has generally been overlooked. Consider Figure 4 of the Appendix, which shows a box-and-whisker plot for the level of trade exports associated with each month. As can be observed, the graph reveals fairly pronounced sinusoidal behavior, with months towards the beginning of the year tending to produce less gross trading exports than those towards the end of the calendar year. The level of trade exports associated with April, May and June, for instance, is much lower than those corresponding to October, November, and December. Such patterns underscore a degree of seasonality within our data set. Up until this point, we have ignored this factor, and our models are consequently inaccurate if we are to take this characteristic into account.

As a result, our models can be revised by still using an ARMA\((p, q)\) process, but incorporating a seasonal factor within it. Such an adjustment does not breach our assumption of stationarity because within the autoregressive model, we are assuming that the epsilons are independent and contain everything not incorporated within the last \(p\) terms. Thus, it can be argued that the seasonality was already lumped within the epsilons and that our data set can be described as seasonally stationary.

An ARMA\((p, q)\) process with seasonality resembles the non-seasonal ARMA\((p, q)\) process described above, as it may contain both an AR and MA component. Its abbreviation is ARMA\((p, q) \times (P, D, Q)\) with period \(s\), where the latter half describes the seasonality of the series. Roughly speaking, this seasonal process can be described by the formula \(y_t = S_t + N_t\), where \(S_t\) is our seasonal component with a period of \(s\) and \(N_t\) is the component of our model that follows a standard ARMA\((p, q)\) process. Unfortunately, obtaining a precise mathematical
formulation of such a process is extremely complex and involves substituting and manipulating several operators that go well beyond the general description outlined above [1, p. 282]. As a result, we shall simply settle for this more intuitive notion of seasonal ARMA models and utilize statistical software packages to perform much of this laborious work.

As compared to a standard ARMA($p,q$) process, selecting the order of the seasonal component of the ARMA($p, q$) x ($P, D, Q$) proves quite straightforward. In this case, $P$ represents the number of seasonal autoregressive terms (SAR), $D$ the number of times seasonal differencing was performed and $Q$ the number of seasonal moving average terms (SMA). Most time series do not have more than one SAR or SMA term, and virtually no data set incorporates more than two. Seasonal differencing should also never be performed more than once.

Figure 3 lists several seasonal ARMA processes, as well as the residual sum of squares associated with each one. Recall that the residual sum of squares is simply the sum of all the differences between the predicted versus the actual observation squared. Therefore although our residual sum of squares looks unconventionally large, it is important to remember that larger observations result in the RSS growing exponentially. For example, if the fitted value differed from the actual value by 1,000 (a difference not terribly unimpressive considering our scale) that difference squared would skyrocket to 1,000,000. Thus given the scale of our data, the higher values associated with the residual sum of squares make the model seem like it is performing much more poorly than it actually is.

Furthermore, Figures 5, 6, 7, 8 and 9 show each of these seasonal ARMA processes versus the raw data. Note that because these processes require evaluating the seasonal data before generating predictive values, most of the seasonal ARMA processes do not begin until either the 14th or 15th observation. In addition, all of these models forecast multiple observations
into the future and are associated with a periodicity of 12 since we are working with monthly data. All graphs of the seasonal ARMA processes were generated through Minitab.

Although observing the residual sum of squares provides an adequate overview, it does not provide the whole picture as to how our model will perform in forecasting future results. In fact, overfitting is a mistake common to time series analysis. Overfitting refers to a model’s tendency to fit a data set so closely, often by including an adundance of terms, that its effectiveness begins to deteriorate during the forecasting process. The residual sum of squares does not take into account the number of terms and simply selecting the model with the least RSS value may encourage such overfitting. One way to evaluate how a model will perform is by analyzing not only how well it “fits” our data, but how well it does at forecasting. Of course, it is impossible to know how well it will forecast future results when we do not know what future values might be. One solution to this predicament is data-splitting or cross-validation. Data-splitting involves dividing our data set into two groups and using the first group to fit the model and the second to analyze how well our model forecasts these observations. Another option, however, is to evaluate the Akaike Information Criterion (AIC), which “punishes” models whose lower RSS values depend on too many terms [1, p. 57]. The AIC can be found through the formula $2k + nlog\left(\frac{RSS}{n}\right)$, where k is the number of terms in the proposed model and n is the number of observations in our series. Consequently, the AIC values associated with ARMA(1,1) x (1,0,1), ARMA(1,0) x (1,0,1), ARMA(0,1) x (1,0,1), ARMA(2,1) x (1,0,1) and ARMA(1,2) x (1,0,1) are 811.6855, 811.38555, 814.41043, 813.54421, and 813.38846 respectively. Such results follow the general trend we would expect with the larger AIC values generally belonging to the models with the most terms and the smaller AIC values corresponding with the models with only fewer terms.
Based on such results, we are inclined to select ARMA(1,1) x (1,0,1) as our model.

Figure 10 of the Appendix shows a 4-1 plot which displays the normal probability plot, residuals versus fitted value, a histogram of the residuals and a time series plot of the residuals. Looking at the normal probability plot, we observe that except for a few values in the upper tail, our data falls tightly around our line of interest and that the histogram produces a bell-curve shape. We are therefore inclined to feel confident about our normality assumption. Furthermore, our plots for the residuals also display no shockingly unusual behavior, enabling us to sufficiently argue that an ARMA(1,1) x (1, 0, 1) process provides an appropriate fit to our data.

However, like most modeling processes, the steps delineated above are not perfect and limitations do exist. One limitation of this project is that no decomposition process was ever employed in hopes of smoothing our data, minimizing the effects of seasonality, and then formulating and comparing the models generated from this revised data set. Adjusting our data set in this manner could potentially result in models that not only better fit historical data, but also produce more accurate forecasts. Another limitation is the statistical software employed. Throughout this process, I primarily used Minitab, while occasionally relying on XLSTAT as needed. XLSTAT has generated less than ideal performance evaluations, and Minitab is limited in the functions it can perform in regards to time series. Many statistical packages such as R, SAS, or STATA could have provided a wider variety of tests and procedures to refine our results, but because they rely on a precise and rather complicated programming language, it was not possible to utilize them due to time constraints.

Due to the pressing applicability of trade exports, there exist many topics of future research that springboard from this project. One such topic involves statistically analyzing and taking into account the countries surrounding Zimbabwe since it is landlocked. In his book, *The
Bottom Billion: Why the Poorest Countries are Failing and What Can Be Done About It, economist Paul Collier describes how landlocked countries face a host of obstacles in exporting products, many of which are completely out of their hands. One such obstacle stems from the fact that a landlocked country’s chief trading partners are its immediate neighbors, and in Zimbabwe’s case, the top two countries to which its goods are exported are South Africa and Mozambique. Zimbabwe’s trade performance consequently hinges heavily upon the political, economic, and social stability of these countries, a scenario that cannot be taken for granted given the more volatile conditions of many African countries. Therefore, if Mozambique’s or South Africa’s economy suffers, then so does Zimbabwe’s. Yet perhaps just as important as the economic conditions of neighboring countries is the fact that landlocked countries do not have direct access to trading ports; they are dependent on their neighbors’ willingness to let them use their ports. They also rely heavily on peaceful stability in neighboring countries, as war or other forms of political instability greatly hinder a landlocked country’s ability to access those port locations [4, p. 55-57].

As a result, simply analyzing an individual country’s trading patterns does not provide a complete picture. More refined models and forecasts can be obtained by incorporating factors relating to the economic and political conditions of Zimbabwe’s neighbors. For example, if Mozambique’s economy is starting to plummet, but that scenario has not yet manifested itself in the raw data of Zimbabwe’s trade exports, then our generated forecasts are useless. Factoring the economic conditions of neighboring countries into our models depends upon multivariate time series analysis and is one of the most revolutionary topics within international development and public policy today. Using time series in this manner would therefore be an excellent form of future research in regards to this project.
Statistical time series affords a powerful means to address some of today’s most pressing issues. In this project, we specifically explored how to analyze, model, and forecast the behavior of stationary time series with some degree of seasonality by using AR, MA and ARMA processes. Future research should focus on developing a multivariate setting in which the economic and political factors of Zimbabwe’s neighbors are taken into account. Such a project both reveals how time series is extremely applicable within a real world setting and demonstrates the rich intersection between statistics and many other fields of social interest. This project consequently underscores why such a partnership should continue to be fostered if we are to tackle some of the issues most relevant to developing countries today.
Appendix

Figure 1

Time Series Plot of Exported Data in U.S. Currency

Figure 2

Autocorrelation Function for Exported Data in U.S. Currency
(with 5% significance limits for the autocorrelations)
Figure 3

Partial Autocorrelation Function for Exported Data in U.S. Currency (with 5% significance limits for the partial autocorrelations)

Partial Autocorrelation vs Lag
<table>
<thead>
<tr>
<th>Model</th>
<th>Residual Sum of Squares</th>
<th>Suggested Model Equation</th>
</tr>
</thead>
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<td>2440275199845</td>
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<tr>
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<td>1569905276669</td>
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<td>MA(5)</td>
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<tr>
<td>AR(2)</td>
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<td>AR(3)</td>
<td>464711595791</td>
<td>$y_t = \varepsilon_t + 0.6486y_{t-1} + 0.1799y_{t-2} + 0.1654y_{t-3}$</td>
</tr>
<tr>
<td>AR(4)</td>
<td>463774600397</td>
<td>$y_t = \varepsilon_t + 0.6561y_{t-1} + 0.1884y_{t-2} + 0.1968y_{t-3} - 0.0483y_{t-4}$</td>
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<tr>
<td>ARMA(1,1)</td>
<td>465860390380</td>
<td>$y_t = \varepsilon_t + 0.9973y_{t-1} + 0.3891\varepsilon_{t-1}$</td>
</tr>
<tr>
<td>ARMA(1,2)</td>
<td>443851826683</td>
<td>$y_t = \varepsilon_t + 0.9998y_{t-1} + 0.5127\varepsilon_{t-1} + 0.4210\varepsilon_{t-2}$</td>
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<tr>
<td>ARMA(1,3)</td>
<td>426555240638</td>
<td>$y_t = \varepsilon_t + 0.9998y_{t-1} + 0.5218\varepsilon_{t-1} + 0.2467\varepsilon_{t-2} + 0.2081\varepsilon_{t-3}$</td>
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<tr>
<td>ARMA(1,4)</td>
<td>382713541463</td>
<td>$y_t = \varepsilon_t + 1.0002y_{t-1} + 0.5719\varepsilon_{t-1} + 0.1536\varepsilon_{t-2} - 0.0237\varepsilon_{t-3} + 0.3367\varepsilon_{t-4}$</td>
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<tr>
<td>ARMA(2,1)</td>
<td>460988254920</td>
<td>$y_t = \varepsilon_t + 1.1725y_{t-1} - 0.1746y_{t-2} + 0.5945\varepsilon_{t-1}$</td>
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<tr>
<td>ARMA(2,2)</td>
<td>460131269649</td>
<td>$y_t = \varepsilon_t + 0.2414y_{t-1} + 0.7546y_{t-2} - 0.3889\varepsilon_{t-1} + 0.3769\varepsilon_{t-2}$</td>
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<tr>
<td>ARMA(2,3)</td>
<td>458225956721</td>
<td>$y_t = \varepsilon_t + 0.1820y_{t-1} + 0.8160y_{t-2} - 0.4285\varepsilon_{t-1} + 0.5193\varepsilon_{t-2} + 0.1298\varepsilon_{t-3}$</td>
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<tr>
<td>ARMA(2,4)</td>
<td>394562175414</td>
<td>$y_t = \varepsilon_t + 1.3890y_{t-1} - 0.3888y_{t-2} + 0.8615\varepsilon_{t-1} - 0.0436\varepsilon_{t-2} - 0.0710\varepsilon_{t-3} + 0.2388\varepsilon_{t-4}$</td>
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<tr>
<td>ARMA(3,1)</td>
<td>460841430723</td>
<td>$y_t = \varepsilon_t + 0.0463y_{t-1} + 0.6054y_{t-2} + 0.3363\varepsilon_{t-1} - 0.6120\varepsilon_{t-2}$</td>
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<tr>
<td>ARMA(3,2)</td>
<td>409240986511</td>
<td>$y_t = \varepsilon_t + 0.7742y_{t-1} + 0.6602y_{t-2} - 0.4349\varepsilon_{t-1} + 0.2093\varepsilon_{t-2} + 0.7680\varepsilon_{t-3}$</td>
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<tr>
<td>ARMA(3,3)</td>
<td>402548436555</td>
<td>$y_t = \varepsilon_t + 1.1898y_{t-1} + 0.1925y_{t-2} - 0.3824y_{t-3} + 0.6082\varepsilon_{t-1} + 0.5138\varepsilon_{t-2} - 0.1099\varepsilon_{t-3}$</td>
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<tr>
<td>ARMA(4,1)</td>
<td>422849414020</td>
<td>$y_t = \varepsilon_t + 1.3163y_{t-1} - 0.1521y_{t-2} - 0.1340y_{t-3} - 0.0298y_{t-4} + 0.9837\varepsilon_{t-1}$</td>
</tr>
<tr>
<td>ARMA(4,2)</td>
<td>409677426437</td>
<td>$y_t = \varepsilon_t + 0.8473y_{t-1} + 0.5629y_{t-2} - 0.1960y_{t-3} - 0.2138y_{t-4} + 0.3331\varepsilon_{t-1} + 0.6363\varepsilon_{t-2}$</td>
</tr>
<tr>
<td>ARMA(4,4)</td>
<td>393389431752</td>
<td>$y_t = \varepsilon_t + 1.3153y_{t-1} - 0.2227y_{t-2} - 0.2600y_{t-3} + 0.1675y_{t-4} + 0.8325\varepsilon_{t-1} + 0.0667\varepsilon_{t-2} - 0.2668\varepsilon_{t-3} + 0.3451\varepsilon_{t-4}$</td>
</tr>
</tbody>
</table>
Table 2

<table>
<thead>
<tr>
<th>Process</th>
<th>Associated Residual Sum of Squares</th>
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<tr>
<td>ARMA(1,1) x (1, 0, 1)</td>
<td>310429803992</td>
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<tr>
<td>ARMA(1,0) x (1, 0, 1)</td>
<td>325328897542</td>
</tr>
<tr>
<td>ARMA(0,1) x (1, 0, 1)</td>
<td>353356458819</td>
</tr>
<tr>
<td>ARMA(2,1) x (1, 0, 1)</td>
<td>309230153950</td>
</tr>
<tr>
<td>ARMA(1,2) x (1, 0, 1)</td>
<td>307912698105</td>
</tr>
</tbody>
</table>
Figure 9

Time Series Plot of Raw Data, ARMA(1,2)x(1,0,1)

Value of Exported Data in U.S. Currency

Month

Figure 10

Residual Plots for Raw Data

Histogram

Normal Probability Plot

Versus Fits

Versus Order

Residual

Fitted Value

Observation Order
References


2. Introduction to Time Series Analysis, National Institute of Standards and Technology, online.


I grant permission to Dr. Lamarr Widmer for the use of my paper “Modeling and Forecasting Zimbabwe’s Trade Exports with Time Series Analysis” for educational purposes.
Emily Smetak, 4/20/2017