

2010

The Development of Mathematical and Spiritual Maturity in the Undergraduate Mathematics Curriculum

Angela Hare
Messiah College

Follow this and additional works at: <https://mosaic.messiah.edu/facscholar>



Part of the [Christianity Commons](#), and the [Higher Education Commons](#)

Permanent URL: <https://mosaic.messiah.edu/facscholar/25>

Recommended Citation

Hare, Angela, "The Development of Mathematical and Spiritual Maturity in the Undergraduate Mathematics Curriculum" (2010). *Faculty Scholarship Papers*. 25.
<https://mosaic.messiah.edu/facscholar/25>

Sharpening Intellect | Deepening Christian Faith | Inspiring Action

Messiah University is a Christian university of the liberal and applied arts and sciences. Our mission is to educate men and women toward maturity of intellect, character and Christian faith in preparation for lives of service, leadership and reconciliation in church and society.

The Development of Mathematical and Spiritual Maturity in the Undergraduate Mathematics Curriculum

Dr. Angela Hare

Associate Professor of Mathematics, Messiah College

August 15, 2009

Introduction:

The application of mathematical tools to human systems has powerfully transformed the ability of human beings to live together peacefully, raise healthy families, and pursue dreams and prosperity rather than mindless toil. Mathematics provides the foundation for the design and construction of structures and programs in all societies, as well as tools for measuring the strength and success of what is built. The work done in the fields of mathematical sciences can serve the poor and promote justice in the world. As just one example, mathematical principles are at the heart of computer science and the broader information and technology revolution of the last century. Technology rests on a mathematical base of logic, and it can give people who are not free the power and will to overcome oppression. Mathematical writer Underwood Dudley claims in fact that mathematics and science are the cure for what ails the writer of the biblical book of Ecclesiastes:

Mathematics and science are the antidotes to the despair that comes from thinking that in spite of mighty efforts nothing changes or will ever change. It is true that in government, in economics, in morals, the wheel of existence goes around and around, endlessly turning, and there is nothing new under the sun . . . around and around the wheel of existence goes. However, it is not true that nothing ever changes and there is nothing new under the sun. In science and mathematics, the arrow points forward only and does not bend into a circle. Whatever happens will be new. It will not be emptiness.¹

Colleges and universities that teach mathematics have a responsibility to develop in students an appreciation of the powerful tools they are studying in the mathematics curriculum. Beyond this fundamental responsibility, the Christian college or university² has the richer task of equipping mathematics graduates to use their knowledge and skills to sharpen their spiritual insight, to serve others, and to promote justice and freedom in society. The growth in

¹ Dudley, Underwood. "Ecclesiastes." In *Readings for Calculus*, edited by Underwood Dudley, pp. 103-105. Washington, DC: Mathematical Association of America, 1993.

² For the remainder of the paper, the author will use 'Christian college' in lieu of 'Christian college or university'.

mathematical maturity that occurs during the undergraduate years is an asset that enables Christian students of mathematics to participate in the redemptive work of Jesus Christ through their discipline of study. The thesis of this paper is that strong reasoning skills developed through mathematical study can inform and deepen students' understanding of scriptural truth and enhance their sense of vocation as a means of stewardship, service, and worship in the kingdom of God. The author begins by illustrating the thesis with the mathematical concept of paradoxes of infinite processes as studied in the undergraduate mathematics curriculum. The paper continues with a broader discussion of a Christ-centered undergraduate education, definitions of mathematical and spiritual maturity, and a model for the vocation of the Christian mathematics educator.

Paradoxes of the infinite

Table 1 lists three examples of paradoxical results students will prove in undergraduate mathematics courses, each dealing with infinite processes. For the purpose of this paper, the author is defining a paradox as an apparently self-contradictory statement that nonetheless contains truth.

<p>1. A set of cubic blocks of dimensions $(1 \times 1 \times 1), (\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}), (\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}), (\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}), \dots$ is infinitely tall when stacked (because $\sum_{n=1}^{\infty} \frac{1}{n}$ is a divergent series) but the set will fit nicely into a small room when tiled on the floor (because $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a convergent series).</p>
<p>2. A region can have an infinite perimeter but a finite area. This means that a painter will never be finished moving his ladder to paint the region in the first quadrant bounded by the x-axis, the line $x=1$, and the graph of $f(x) = \frac{1}{x^2}$, but he will need only a finite amount of paint.</p>
<p>3. There are as many even integers as there are integers, because each even integer $2n$ can be placed in a slot numbered n, without exhausting the correspondence.</p>

Table 1
Paradoxes of the Infinite

A second semester calculus student typically struggles to grasp the truth of the results in Table 1. Throughout the undergraduate mathematics curriculum, students use infinite processes, such as definite integration, to attain finite results, and their mathematical understanding of the infinite grows. A strong fourth-year student is comfortable with the results in Table 1 and with the arithmetic of infinite processes, even if she is still intrigued by them. Between the second and fourth year of mathematical study, the student has progressed in mathematical maturity.

Paradoxes such as those in Table 1 are useful tools in mathematics education because studying them reveals areas of knowledge fragility and provides a powerful motivator (resolving the paradox) that changes knowledge frameworks to the next, more mature, stage.³ In a 2008 study of the mathematical understanding by grade-school children of the concept of infinity, researchers categorized the understanding of infinity as processional, topological, and spiritual, as described in Table 2.

Category*	Example
Processional	“Infinity is something that never ends.” Grade 8 student
Topological	“Something enormous . . . Big . . . large . . . very big.” Grade 8 student
Spiritual	“Infinity is something that has a secret we cannot grasp. We can’t get this secret without the help of God.” Grade 4 student

Table 2
Understanding of Infinity by Grade School Children⁴

*Categories are descriptive, not sequential

³ Moshovitz-Hadar, Nitsha and Rina Hadass. “Preservice Education of Math Teachers Using Paradoxes.” *Educational Studies in Mathematics* 21 (June 1990): pp. 279,284.

⁴ Singer, Florence M. and Cristian Voica. “Between Perception and intuition: Learning about infinity”. *Journal of Mathematical Behavior* 27 (September 2008): pp. 191.

The researchers found that no child was unable to express his/her own ideas about infinity and that “the spiritual dimension is spontaneously expressed by children within discussions about infinity.”⁴ Remembering that this was a study of children’s mathematical understanding, not spirituality, it is interesting to notice the connection between mathematical and spiritual thinking.

Paradoxes of a different type play a similar role in Christian teaching. A common theme in the New Testament is Christ’s use of paradoxes to illustrate the truths of the kingdom of God. Here are a few examples from the gospels:

- “Blessed are you when people revile you and persecute you and utter all kinds of evil against you falsely on my account.” (Matthew 5:11)⁵
- “Those who find their life will lose it, and those who lose their life for my sake will find it.” (Matthew 10:39)
- “But many who are first will be last, and the last will be first.” (Mark 19:30)

In *The Lion and the Lamb: Paradoxes of the Christian Faith*, Gerald Kennedy writes:

*The Christianity which is truly realistic is not afraid of paradox and sets forth its truth in paradoxes . . . Christianity suggests a way of living that often seems contrary to logic. It is not something you could have guessed. It is not something reducible to reason. It breaks in. It startles. It overwhelms our carefulness. It opens a vista so breath-taking that we can hardly stretch our minds and spirits wide enough to take it in.*⁶

Paradoxes of the Christian faith are challenging and mind-stretching, and they can play a role in stimulating spiritual growth that is not unlike the role of paradoxes in mathematics education. How might the growth in mathematical understanding stimulated by the paradoxes of the infinite contribute to a deeper understanding of the nature of God through Christ?

The biblical book of Ecclesiastes tells us “He has made everything beautiful in its time. He has also set eternity in the hearts of men; yet they cannot fathom what God has done from beginning to end” (3:11, NIV). One of the images that God has placed in the hearts of men is that of an infinite, eternal kingdom. Author C.S. Lewis alludes to the heavenly realm using

⁵ Unless otherwise noted, all biblical references are taken from the New Revised Standard Version.

⁶ Kennedy, Gerald. *The Lion and the Lamb: Paradoxes of the Christian Faith*. Nashville, TN: Abingdon-Cokesbury Press, 1950, pp. 18,21.

images and metaphors which include the same infinite processes studied in mathematics. An example can be seen in one excerpt of *Voyage of the Dawn Treader*, one book in Lewis' series *Chronicles of Narnia*. In the excerpt that follows, a boy named Eustace has been enchanted and turned into a dragon because of his greed and selfishness. He can only be restored by the lion, representing Christ. In a clear allegorical scene of baptism and redemption, the lion leads Eustace to a large well, where Eustace is to bathe.

The water was clear as anything and I thought if I could get in there and bathe it would ease the pain in my leg. But the lion told me I must undress first . . . So I started scratching myself and my scales began coming off all over the place. And then I scratched a little deeper, and, instead of just scales coming off here and there, my whole skin started peeling off beautifully . . . I just stepped out of it . . . so I started to go down into the well for my bath. But just as I was going to put my foot into the water I looked down and saw that it was all hard and rough and wrinkled and scaly just as it had been before . . . So I scratched and tore again and this under skin peeled off beautifully and out I stepped and left it beside the other one and went down to the well for my bath.

Well, exactly the same thing happened again. And I thought to myself, 'oh dear, however many skins have I got to take off?' For I was longing to bathe my leg. So I scratched away for the third time and got off a third skin, just like the two others and stepped out of it. But as soon as I looked at myself in the water I knew it had been no good.

Then the lion said . . . "You will have to let me undress you."⁷

The image Lewis creates in this excerpt clear; Christ bridges the gap between the finite and the infinite. Although human sinfulness is infinitely-layered, redemption is not out of reach.

Recall the quote from Table 2, "Infinity is something that has a secret we cannot grasp. We can't get this secret without the help of God." The student who has worked with the infinite in mathematics has enhanced imagery in place to understand and appreciate God's infinite nature and redemptive power. Such a student has developed maturity to accept the truth within mathematical paradoxes, he has seen truth set forth in paradox, and he has a foundation to integrate this maturity with spiritual understanding. When he reads in Matthew 10, "Those who find their life will lose it, and those who lose their life for my sake will find it", he can step out in faith and take actions that may require 'losing' his life, with the understanding that paradoxical statements can be true. Indeed, they can be proven to be true and counted on. A mathematical understanding of the infinite and eternal can strengthen spiritual understanding.

⁷ Lewis, Clives S. *Voyage of the Dawn Treader, Book 5 in the Chronicles of Narnia*. New York: HarperCollins, 2000: pp. 107,108. (Reprinting of original 1952 edition).

A Context for Christ-centered Undergraduate Education

In *Conceiving the Christian College*, Dr. Duane Litfin, president of Wheaton College, maintains that a Christian college should be distinctly known for placing the person of Jesus Christ at the center of its mission, with his central claim on the lives of his followers evident throughout the programs of study.⁸ Litfin admits that this mandate for Christian undergraduate education is a strong one, but he argues that it is the calling of the gospel and cannot be dismissed. He writes, “[A Christ-centered education] posits the Son, the Second Person of the Godhead, as the unifying key to all that humans can know or understand, such that in the end nothing can be adequately grasped apart from him. This is an astonishing claim, one that seems too much even for some Christian educators to swallow.”⁹

How is Jesus Christ the unifying key and the center of the pursuit of the curricular goals of a Christian undergraduate mathematics program? The author maintains that an undergraduate mathematics curriculum bears witness to the lordship of Jesus Christ when educators pursue curricular goals that develop both mature mathematical skills and a mature spiritual perspective, teaching students to recognize the eternal truth revealed through mathematical study and nurturing their calling to live and work as members of a human community that is being transformed through Christ. Graduates with these types of maturity will faithfully step into roles in church and society in which they use their mathematical gifts as participants in Christ’s redemption of the world through creative work that is pleasing to him, serving the poor and promoting justice as biblical mandates.

What types of roles will these be? There are many possibilities. Missionaries and professionals in economic development and other social services will attest to the tremendous difference technological advances have made in their work. Bible translators use computer software that has reduced the time it takes to develop an alphabet and syntax for a spoken language from decades to years. Dictators find it increasingly difficult to maintain power as the internet thwarts their propaganda and is difficult to censor. An article in the *Economist* in 2004 describes how cell phones enable people in developing countries to start and run their own businesses, without needing to depend on the unreliable communications infrastructure of their country.¹⁰ Applied mathematicians design efficient systems and structures that maximize profit, minimize waste, and in some cases save human lives. Mathematics teachers of children and

⁸ Litfin, Duane. *Conceiving the Christian College*. Grand Rapids: William B. Eerdmans, 2004.

⁹ Litfin, p. 37.

¹⁰ “Beyond the Digital Divide.” *The Economist* 370 (March 11, 2004).

adults equip people to step out of deprivation and into meaningful work. Students of mathematics who are educated with the thesis of this paper in mind will have a solid mathematical foundation, as well as the strong undertow of Christ's call on their lives, drawing them toward redemptive change in the world.

Mathematical and Spiritual Maturity Defined

Surprisingly, there is no consensus in the mathematics education literature about what it means for a college graduate to be mathematically mature. The author could find no general definition of mathematical maturity in current or historic literature in the field. Professional organizations such as the Mathematical Association of America publish recommendations for strong undergraduate programs,¹¹ and one can surmise that a graduate is mathematically mature if she excels in a program that follows these recommendations with rigorous standards, and if she is well-prepared for employment or further study in related fields. The author proposes the following definition of mathematical maturity, given in Table 3:

A student attains optimal mathematical maturity as an undergraduate in a mathematics major if he

- excels in a quality program of college or university mathematical study as measured by professional standards.
- is recognized by faculty and employers at the time of graduation as being prepared for further independent study in mathematical science fields.

Table 3
Working Definition of Mathematical Maturity

In terms of Christian faith development, study of the biblical narrative reveals that the indicators of spiritual maturity are obedient steps of Christian faith that are rooted in biblical knowledge. In the field of Christian spiritual development, there exist a variety of statistically reliable and valid scales that measure spiritual maturity, although not in the biblical sense indicated in the previous sentence. These scales include the Intrinsic Religious Orientation

¹¹ Committee on the Undergraduate Program in Mathematics (CUPM). Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004. Washington, DC: Mathematical Association of America, 2004.

Scale,¹² the Christian Religious Internalization Scale,¹³ and the Spiritual Experience Index.¹⁴ Each of these scales measures, among other factors, a person's degree of internalization of religious faith; that is, the degree to which he has transformed external regulations or values into internal ones.¹⁵ The literature consistently links spiritual maturity with both a high level of internalization and also an open and inclusive view of complex faith issues (i.e., the opposite of dogmatism).

The weakness of these indices in the literature, in the author's opinion, is the lack of a biblical and Christ-centered basis. The concepts of internalization and openness would be expected characteristics of one's mature perspective in many areas of life other than Christian spirituality. One would expect, for example, that a mature political view or a mature appreciation of music would include both strong internal support and openness to complex ideas. These characteristics, while worthwhile, provide rather weak and general goals for the Christian college educator in terms of spiritual development. The author prefers a working definition of Christian maturity that emphasizes obedience to the mandates of Christ, with the biblical virtues of Christian love and other fruits of the Holy Spirit mitigating the dogmatic insensitivity that perhaps academics address with the concept of spiritual openness.

The author proposes a working definition of Christian spiritual maturity, for the purposes of this paper, as given in Table 4:

<p>A Christian undergraduate student is spiritually mature if she</p> <ul style="list-style-type: none">• articulates a foundational understanding of Scripture and a commitment to biblical truth.• steps into roles in church and society that fulfill biblical mandates for Christian worship and service.• exhibits the fruits of the Holy Spirit in her relationships.

Table 4
Working Definition of Christian Spiritual Maturity

¹² Allport, Gordon W. and J. Michael Ross. "Personal religious orientation and prejudice". *Journal of Personality and Social Psychology* 5 (April 1967): pp. 432 – 443.

¹³ Ryan, Richard M., Scott Rigby and Kristi King. "Two types of religious internalization and their relations to religious orientations and mental health." *Journal of Personality and Social Psychology* 65 (September 1993): pp. 586-596.

¹⁴ Genia, Vicky. "The Spiritual Experience Index: Revision and Reformulation." *Review of Religious Research* 38 (September 1996): pp. 344-361.

¹⁵ Welch, Ronald and Kimberlee Mellberg. "Spiritual Maturation and Religious Behaviors in Christian University Students". *Christian Higher Education* 7 (April 2008): pp. 145.

These definitions will serve as benchmarks in the following discussion of a model for the Christian college mathematics educator.

A Model for the Vocation of the Christian College Mathematics Educator

To develop a useful model for the vocation of a mathematics educator at a Christian college, one might begin with something like what is shown in Figure 1. This model indicates two developmental aspects that are of interest to mathematics faculty at a Christian college. The first aspect, shown on the horizontal axis in the figure, is the growth in mathematical understanding that occurs during the undergraduate years. The second is the development of maturity in students' Christian faith, equipping them to step confidently into roles of service or leadership. This aspect is shown by the vertical axis in Figure 1.

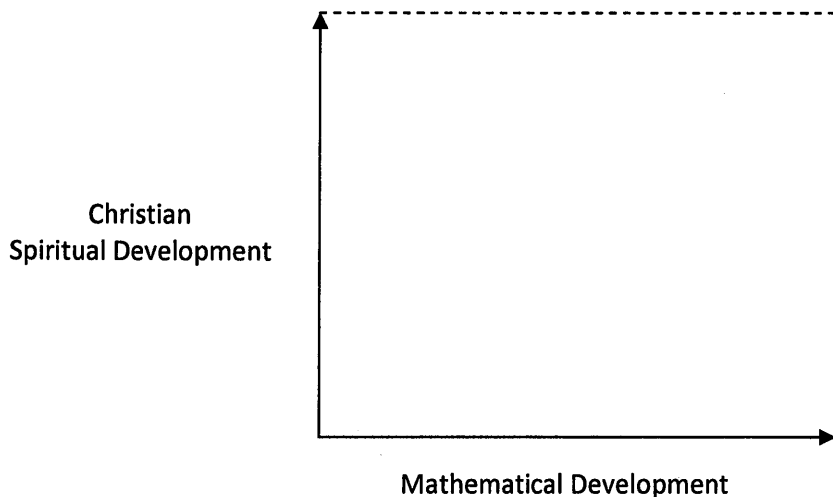


Figure 1

A Model for Student Growth in Undergraduate Mathematics at a Christian College

Students begin their years at a Christian college at various starting points along the two continua shown in Figure 1, and they attain differing amounts of growth during their 4+ years of the undergraduate experience, as illustrated in Figure 2.

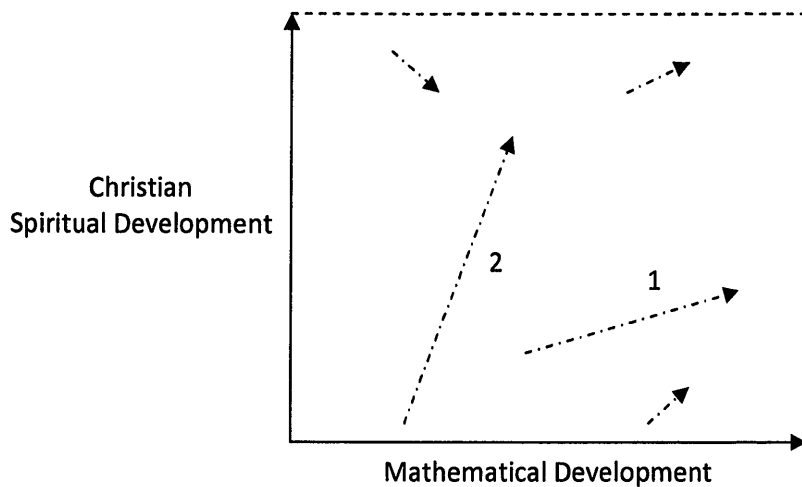


Figure 2

An Illustration of Student (Math Major) Growth in the Undergraduate Years

Each dashed arrow in the interior of Figure 2 indicates the growth of a hypothetical student majoring in mathematics at a Christian college, over the 4+ year span of their college education. As an example, Student 1 in the figure achieves significant maturity in mathematical understanding but minimal spiritual growth, while Student 2 graduates with a modest level of mathematical maturity while attaining more spiritual growth than any other student represented in the model.

As with all concrete models, there are flaws with the model presented in Figures 1 and 2. The model lacks measurable milestones along the continua of spiritual and mathematical development. The working definitions of both mathematical and spiritual maturity given in Tables 3 and 4 suggest guidelines for developing these milestones. The model does illustrate that, within the mathematics major curriculum, the Christian mathematics educator seeks to teach and guide her students in such a way that they develop mathematical and spiritual understanding, not in isolation of each other, but as complementary aspects of a growth process that leads to a firmer grasp of Christian truth. A mathematics professor at a Christian college who teaches with only the horizontal axis in mind is not doing his job well. In this way, faculty members of Christian colleges have a richer, and more demanding, task than colleagues at secular institutions. The earlier discussion of the concepts of paradoxes of the infinite is illustrative of the process by which a professor of mathematics at a Christian college can teach and mentor

students according to this model. By teaching students about the mathematics of infinite processes with reference to spiritual insight about the eternal, a professor helps students to build spiritual understanding that is not fragile or dogmatic. It will be internalized knowledge, with a multidisciplinary framework, and this understanding can withstand the challenge of complex ideas. These are characteristics of a mature Christian faith. It is significant that only at Christian schools do professors have the liberty, and responsibility, to integrate the development of spiritual insight and mathematical understanding in their teaching and mentoring roles.

The author is not suggesting that mathematical understanding at the undergraduate level is necessary for comprehending Christian truth. Other models may exist in which Christian spiritual development is attained through study of the arts or through athletics, but these models are not under consideration in this paper. Nor is the author claiming that the overall development of an undergraduate student studying mathematics as a major can be isolated into only two dimensions. The model in Figure 2 simply highlights in a useful way the integrated maturation process that the Christian mathematics educator should be guiding. The reasoning skills developed through mathematical study (one continuum in the model) can inform and deepen students' understanding of scriptural truth and enhance their sense of vocation as a means of stewardship, service, and worship in the kingdom of God (a second continuum). Mathematical study at the undergraduate level can inform and deepen students' understanding of scriptural truth.

Conclusion

“Mathematics and science are the antidotes to the despair that comes from thinking that in spite of mighty efforts nothing changes or will ever change.” Underwood Dudley believes that what was lacking in the society in which the writer of Ecclesiastes expresses his despair was science and mathematics.¹⁶ It is true that the mathematical sciences have propelled human progress in remarkable ways, particularly since the discovery of calculus. It is a worthy but challenging goal for mathematics educators at Christian colleges and universities of the liberal and applied arts to teach these powerful disciplines with a view toward the type of maturity described in this paper. Such a goal is in line with the mission of the author's institution, “to educate men and women toward maturity of intellect, character, and Christian faith in preparation for lives of service, leadership, and reconciliation in church and society.”¹⁷ In 2 Timothy 2:15, we read “Do your best to present yourself to God as one approved by him, a

¹⁶ Dudley, Underwood. “Ecclesiastes.” In *Readings for Calculus*, edited by Underwood Dudley. Washington, DC: Mathematical Association of America, 1993, pp. 104.

¹⁷ Messiah College catalog, 2008-2009, pp. 6.

worker who has no need to be ashamed, rightly explaining the word of truth.” Truth has many forms and expressions, including mathematical ideas and reasoning skills, which are tools in the Christian pursuit of biblical ideals. The student of mathematics at a Christian college should see herself, within her major, as a member of the body of Christ, blessed with certain gifts and training and equipped to participate personally and professionally in the broader community of Christian faith. Prepared in this way, mature Christian students of mathematics will be ready to move confidently, even during their undergraduate years, into positions in which the abilities honed in their major are recognized as factors that add depth and insight to their faith in Christ and to projects and organizations that promote a Christ-centered vision and hope.